CHAPTER I

PRINCIPLES OF ELECTRICITY

1.1. -ELECTRIC ENERGY

The electric energy is one of the forms the energy may appear. It may be obtained using different principles. The equipment used for the generation could be divided in a general way as batteries or electric generators.

If the attention is concentrated in the used energy in residences, commerce or industries, then this electric energy is generally obtained from generation stations and transmitted to the users through electric lines and transformers. The generation stations may use different principles for the conversion of energy into electric energy it self. If it is converted chemical energy, obtained from petroleum or other materials, then the station is a thermoelectric. When it is converted the mechanical energy from water falls, the process is done by a hydroelectric. If the source of energy is the atomic nucleus, then the electric energy is obtained from a nuclear station. Electric energy may be also generated using other sources as the wind, the sea tides, etc, but the three enumerated before are the more frequently used.

The generated energy per unit time is known as power, and measured in watts (W). For the generated energy the multiples kilowatt (1 kW = 1000 W) or megawatt (1 MW = 10^6 W) are commonly used. There are two parameters that basically describe the transmitted power, they are:

- The electric current intensity, which is the quantity of electric charges that circulate across any material per unit time, expressed in Amperes (Amp).
- The electromotive force (EMF), which is the quantity of energy per unit electric charge, and expressed in Volts (V). When dealing with the loss of energy per unit charge in any part of the electric circuit, it is used the term voltage, also measured in volts.

1.2. –BASIC CIRCUIT COMPONENTS

As a beginning, let’s present three important elements that are going to be encountered in electrical circuits: the resistor, the inductor, and the capacitor. Their circuit symbols are presented in Figure 1.1.

Figure 1.1. Circuit symbols for resistor, inductor, and capacitor.

The resistance is the opposition presented by any material to the circulation of electric charges through it. A resistor may appear as a cylindrical piece of solid material. Inductors consist of isolated wire windings used among other equipment in electric motors, generators and transformers. Capacitors may be structured as layers of metallic foil separated by an insulating sheet of paper and rolled up in a cylinder.

Resistance

The voltage and the current are related in a resistor by the Ohm's Law

$$v = iR$$

Where v is the voltage across the resistor, i is the electric current in the resistor, and R is the resistance, measured in ohms (W).

The mathematical relation among power, current and voltage is presented is a general way for a resistive circuit as

$$p = vi$$

Where p is the power, v the voltage and i the electric current intensity or simply the current.

Capacitance

For any device, having the ability of storing electric energy, there is a relation between the electric charge (q) and the voltage applied between two terminals given by

$$C = q/v$$

C is called the capacitance, measured in farads (F). As the farad is a too big unit, the practical units are submultiples like the microfarad (1 mF = 10^{-6} F) and smaller. The capacitors are used for starting some types of motors, changing the power factor, etc.

Inductance

When the electric current is not constant, but varies with the time, there appears an induced voltage, proportional to the current variation with the time (di/dt). The proportionality constant is known as inductance (L), and measured in henry (H). The expression that relates these parameters is

$$V = L\ (di/dt)$$

This effect is particularly notable when a wire is winded forming a coil. The possibility of inducing a voltage when the current varies with the time has an extraordinary importance, among other reasons, because thanks to this property devices like transformers and others are constructed.

1.3. -CIRCUIT CALCULATIONS: THE OHM’S LAW

The electric current generated by batteries does not vary with the time, as shown in Figure 1.2 a) and is known as direct current (DC). Current generated by other means, for example electric generators vary with the time and frequently has the shape shown in Figure 1.2 b) This is known as alternating current (AC).

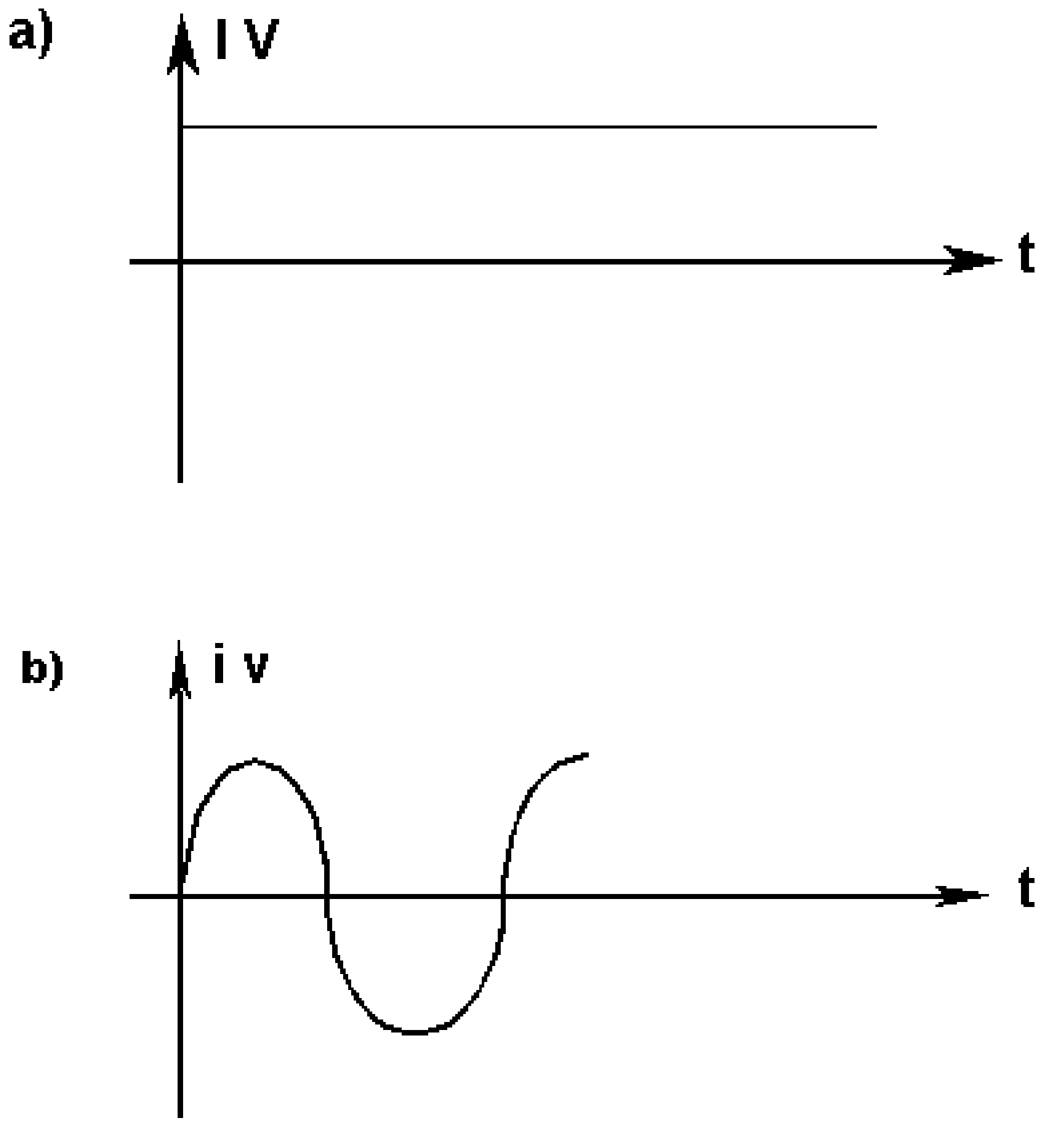


Figure 1.2 a) Direct Current, b) Alternating Current

The Ohm's Law is true for both DC and AC, but in order to simplify the explanation, the following paragraphs will deal only with direct current. When dealing with DC it is usual to represent the voltage, current and power with capital letters: V, I, and P. The battery with its EMF is represented by the symbol in Figure 1.3.

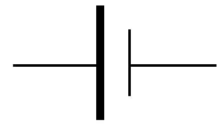


Figure 1.3. Elementary Battery.

For the elementary circuit shown in figure 1.4 , the current that circulate due to the battery EMF is calculated using the expression 1.

I = V/R = 12/4 = 3 Amp.

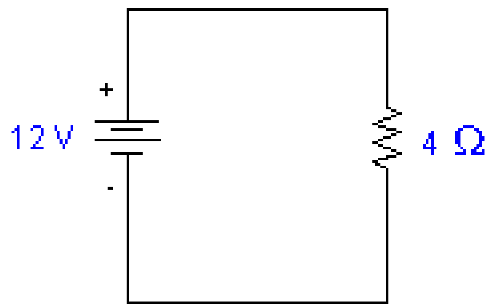


Figure 1.4. Basic Circuit.

Series Circuit

Two or more circuit elements are said to be in series if the same current flows through each of the elements. The circuit in Figure 1.5 is an example of a series circuit.

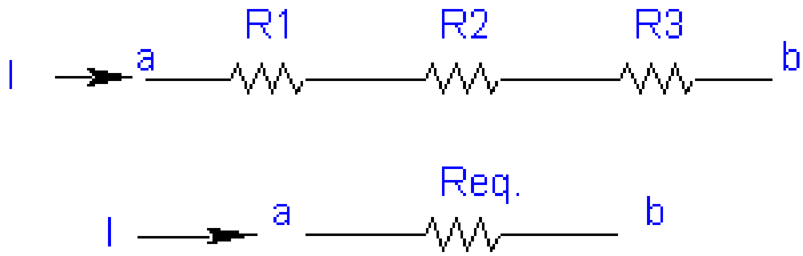


Figure 1.5. Resistors R_1, R_2, and R_3 connected in series

The resistors in Figure 1.5 may be replaced, for circuit calculations, by one single resistor R_{eq}. This will be the equivalent resistance that the battery "sees", or in other words, the resistance that connected in the circuit between terminals a and b, does not affect the voltage drop and the current that circulates between these two terminals. R_{eq} can be calculated for this example under the following two conditions of the series circuit:

- The voltage drop between terminals a and b is the sum of the voltage drops V_1, V_2, and V_3 across each resistance: $V = V_1 + V_2 + V_3$
- The current is the same for all the resistances.

From the Ohm's law: $V = IR$

$$V = IR_1 + IR_2 + IR_3 = I\,(R_1 + R_2 + R_3\,) = IR_{eq},$$

$$R_{eq} = R_1 + R_2 + R_3$$

Example No. 1. 1:

For the circuit shown in Figure 1.6, find the current that circulates and the voltage drop across the 10 Ohms resistance.

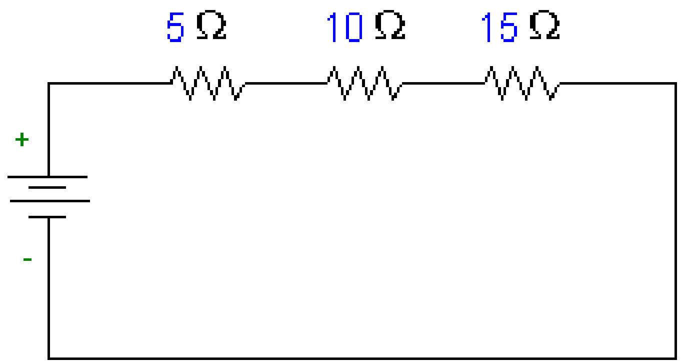


Figure 1.6. Resistors in series

$R_{eq} = 5 + 10 + 15 = 30\ \Omega$;

$I = 60/30 = 2$ Amp;

$V_{10} = 2 \times 10 = 20$ V.

Parallel Circuit

Two or more circuit elements are said to be in parallel if the same voltage appears across each of the elements. In general, the loads in any electrical energy feeder are going to be connected in parallel. The circuit in Figure 1.7 is an example of a parallel circuit.

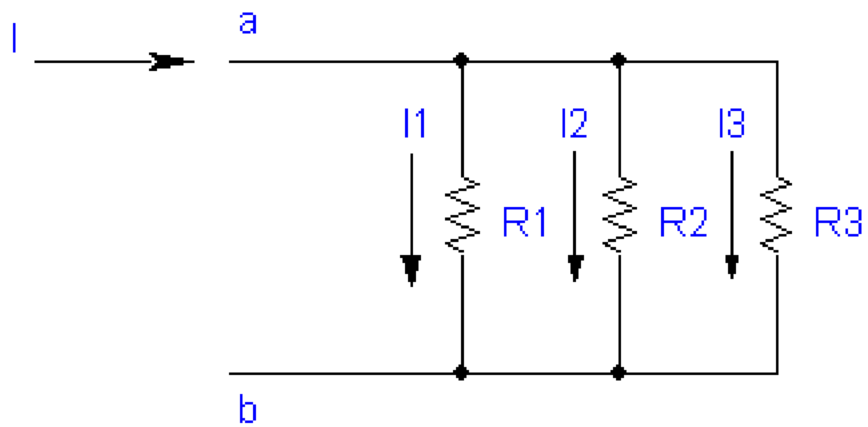


Figure 1.7. Resistors R_1, R_2, and R_3 connected in parallel

The equivalent resistance in this case can be calculated from the following two conditions:

- The current that enters terminal a is the sum of the currents that pass through each resistance.
- The voltage drop between terminals a and b is the same for each resistance.

$$I = I_1 + I_2 + I_3 = V/R_1 + V/R_2 + V/R_3 = V(1/R_1 + 1/R_2 + 1/R_3)$$

$$R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$$

Example No. 1. 2:

For the circuit shown in Figure 1.8, find the current that goes out from the battery and the current that circulates across the 3 Ohms resistance.

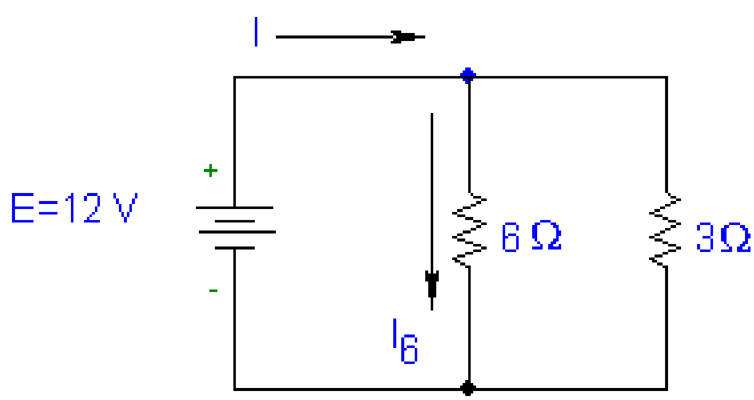


Figure 1.8. Resistors in parallel

$R_{eq} = 1/R_1 + 1/R_2 = 1/6 + 1/3 = (6x3)/(6 + 3) = 2\ \Omega$

$I = 6$ amp

$I_6 = 12/3 = 4$ Amp.

1.4. RESISTANCE AND RESISTIVITY

The resistance of different materials vary very much. Some, such as the metals, are good conductors of electricity. Others, such as rubber, porcelain, etc are called insulators because they practically do not conduct the electrical current.

As can be easily understood, the resistance of a conductor depends not only upon the material of the conductor but also upon the conductor's dimensions and the distribution of the current throughout the cross section of the conductor. For the case under study a uniformly distributed current throughout the cross section of the conductor will be assumed.

For a piece of conductor as the presented in Figure 1.9, the resistance will respond to the relation

$$R = \rho\, l/A,$$

Where l is the conductor length, A is the area of the cross section and r is the resistance of the unit length, unit area conductor, known as resistivity.

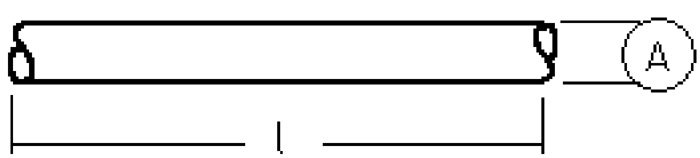


Figure 1.9 . Conductor with length l and cross section A

As the cross section of the conductors is relatively small for the defined units, it is useful to define some unit to be used in this and other similar cases. This unit is the **circular mil (cmil)**, defined as the area of a circle 1/1000 inches in diameter (see Figure 1.10). The unit for the resistivity can be extracted from the definition as (Ohm x cmil)/foot.

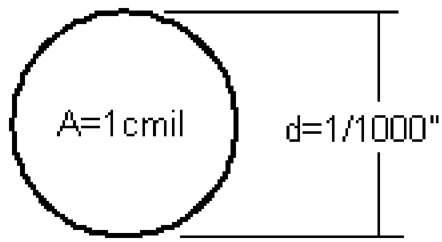


Figure 1.10. Circular mil

Example No. 1. 3.

The resistance of 1 cmil.ft (A = 1 cmil, l = 1 foot) of cupper at 23º C (75º F) is 10.5 Ω. What is the resistance of 600 ft of cupper wire 0.021 in. in diameter?

ρ = 10.5 (Ohm x cmil)/foot

$R = (10.5 \times 600)/ \Pi d^2/4$

As it is known, $\Pi/4 = 0.785$,

$R = 6{,}300/0.785d^2 = 8{,}025.48/21^2 = 18.2\ \Omega$

1.5. CHANGE OF RESISTANCE WITH CHANGE OF TEMPERATURE

The resistance of all pure metals increase with the increment in temperature. The proportion that resistance changes per degree rise in temperature is called the temperature coefficient of resistance. For all pure metals, the coefficient is practically the same: 0.004 for temperatures in degrees Celsius and 0.0023 for temperatures in degrees Fahrenheit. The formula for the calculation is

$$R_h = R_c + aR_c(T_h – T_c),$$

Where R_h is the resistance in Ohms, hot, R_c is the resistance in Ohms, cold; T_h and T_c are the temperatures hot and cold in degrees; and a is the temperature coefficient of the conductor.

Example No. 1. 4.

The resistance of 1 cmil.ft of cupper is 9.59 at 32º F. What will its resistance be at 75º F?

$R_h = 9.59 + 0.0023 \times 9.59 (75 – 32) = 10.51\ \Omega$

1.6. FUNDAMENTALS OF ALTERNATING CURRENT

Most electrical power systems supply alternating current (AC). Typically, the current and voltage in an AC circuit have the shape of a sine (or cosine) wave with respect to time and are expressed as

$$i = I_{max} \sin \Theta \quad \text{and} \quad v = V_{max} \sin \Theta$$

where i, v are the instantaneous values of current and voltage respectively for any value of the angle Q, and I_{max}, V_{max} are the maximum values of current and voltage respectively. (See Figure 1.11).

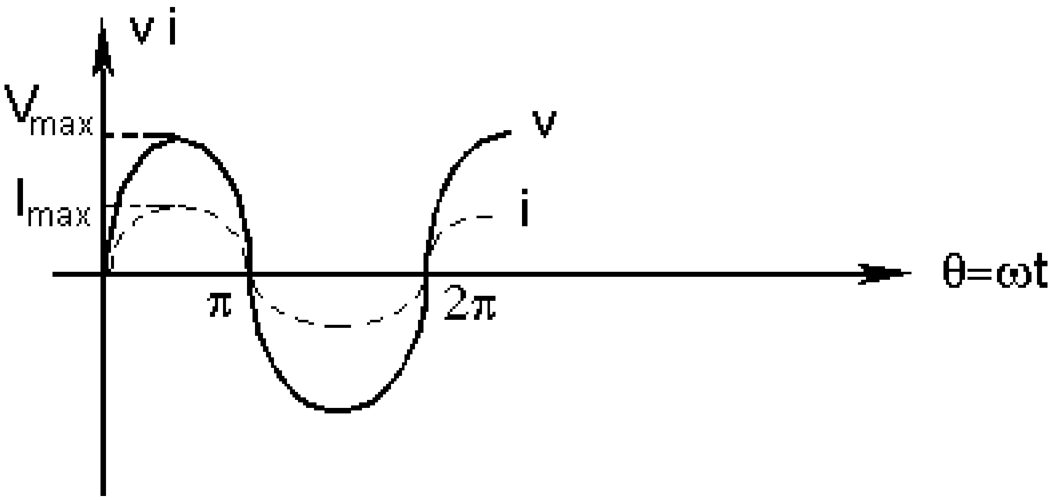


Figure 1.11. Alternating voltage and current

As the electric wave is going to be represented in the time domain, it is useful to express it as a function of time using the relation

$$\Theta = \omega t$$

where w represent the angular frequency in radians per second (rad/sec) and t is the time. Figure 1. shows the two waves, in phase, obtained from the projection of a rotating vector (fasor) on the vertical axis for different values of the angle Q = wt. In this particular case, the initial time has been selected coincident with Q = 0, but in general it is possible to select the initial time for the sinusoidal wave for any position j of the fasor and then the equations will be given by

$$i = I_{max} \sin(\omega t + \phi) \quad \text{and} \quad v = V_{max} \sin(\omega t + \phi)$$

Functions that behaves in the related manner are known as circular functions and it is useful to speak of such functions as having a frequency of f cycles per second (or Hertz). This means that the function is repeated f times each second. The time for one cycle is known as the function period T = 1/f, which occurs at the point marked wt = 2p in Figure 1. B). Therefore

$$T = 1/f = 2\pi/\omega$$

Or as frequently expressed, in the form

$$\omega = 2\pi f.$$

The previously presented Ohm's Law is also applicable to the AC circuits. The basic difference is given by the fact that in AC the mathematical equations are more complicated because the voltage and current values are not constant with the time. Also the introduction of inductance and capacitance in the AC calculations complicate the process.

In order to simplify the calculations, it is recommendable to work whenever possible not with the instant values of current and voltage, but with their "effective (eff)" or "Root Mean Square (RMS)" value. For sinusoidal waves, this value is given by

$$I_{RMS} = I_{max}/\sqrt{2} \quad \text{and} \quad V_{RMS} = V_{max}/\sqrt{2}$$

The reason for using the RMS value is because if applying an AC voltage of V_{max} Volts to the terminals of a resistor, the value of the dissipated power will be the same to the obtained applying a DC voltage of value $V_{max}/\sqrt{2} = V_{RMS}$ to the same resistor. This is the reason of the name "effective".

Example No. 1. 5.

The voltage used for residence lighting is given by the relation $v = 169 \sin 377t$. From this equation, calculate:

a. V_{max}

b. V_{RMS} or V_{eff}

c. Frequency in cycles per second (cps)

d. I_{max} across a 10 W resistor

e. I_{RMS} across a 10 W resistor

Answers

a. V_{max} = 169 Volts

b. V_{RMS} = 169/√2 = 120 Volts

c. f = 377/2p = 60 cps

d. $I_{max} = V_{max}/R$ = 169/10 = 16.9 Amps

e. $I_{RMS} = V_{RMS}/R$ =120/10 = 12 Amps

1.7. –POWER IN AC CIRCUITS

The instantaneous power dissipated by a circuit element is given by the product of the instantaneous voltage and current. In the general case, if the voltage and current delivered to an arbitrary load are given by:

$$v(t) = V_{max} \sin(\omega t + \phi_v)$$

$$i(t) = I_{max} \sin(\omega t + \phi_i)$$

where j_v and j_I are the respective phase angles. The instantaneous power is given by

$$p(t) = v(t)i(t) = [\, V_{max} \sin(\omega t + \phi_v)][\, I_{max} \sin(\omega t + \phi_i)]$$

the previous expression for the power is not easy to handle mathematically. For this reason it is more practical to speak about average power, which can be obtained integrating the instantaneous power over one cycle of the sinusoidal signal. The average power dissipated in a resistive load is obtained from the effective voltage and current, applying the basic power definition as

$$P_{av} = V_{RMS} I_{RMS}$$

When the load is not pure resistive the problem becomes a little more complicated. It is necessary to utilize the phasor representation of voltages and currents. Figure 1.12. shows the effect of a resistor, an inductance, and a capacitor over the current that circulates and the voltage applied through each device.

As can be seen in a), the pure resistive element does not affect the phase angle. If voltage and current were applied in phase, they will remain in phase. The pure inductive circuit shown in b), “moves” the voltage phase ahead with respect to the current (the voltage leads the current); and the pure capacitive circuit shown in c), produces the opposite situation, the voltage lags the current. All these effects have their mathematical explanation, which the interested reader may find in the references at the end of the chapter.

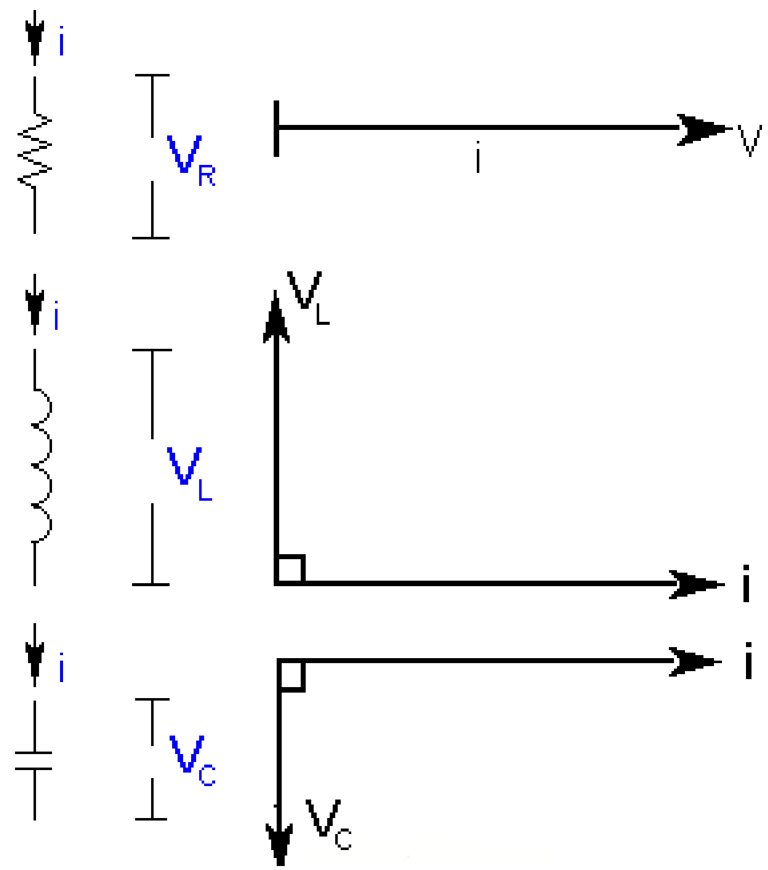


Figure 1.12. Effect of the load over the relative voltage and current phase angles.

a) Pure resistive load

b) Pure inductive load

c) Pure capacitive load

The opposition presented by inductive and capacitive circuits to the pass of the electric current is known as inductive reactance and capacitive reactance respectively and are measured in Ohms (Ω). The formula for it calculation is:

Inductive reactance: $X_L = \omega L$

Capacitive reactance: $X_C = 1/\omega C$

Where L is the circuit inductance in Henry, C the capacitance, in Farad, and ω is the sinusoidal wave angular frequency.

Example 1.6

For the circuit shown in Figure 1.13, calculate the current that circulates if the electromotive force generated by the generator G is given by $v = 141 \sin 377t$.

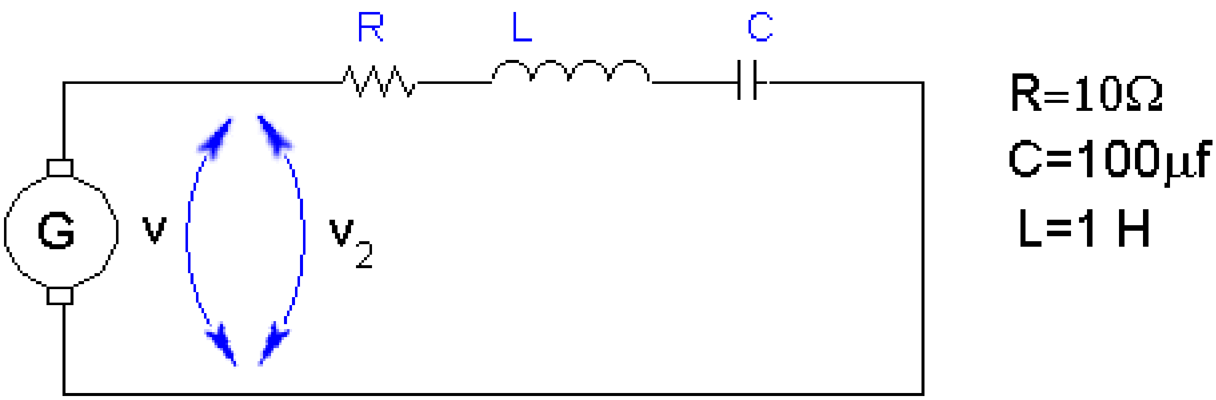


Figure 1.13. Series RLC circuit

In this case ω = 377 rad/sec,

$X_L = 377 \times 1 = 377\ \Omega$

$X_C = 1/(377 \times 10^{-4}) = 26.5\ \Omega$

$X = X_L – X_C = 350.5\ \Omega$

The RLC circuit can be substituted by an equivalent element known as impedance (Z). Figure 1.14 shows in a) the phasor diagram for each element, and for the calculated impedance in b). Note that the calculations are made using simple rules of vector addition. The numerical value of the impedance is calculated as

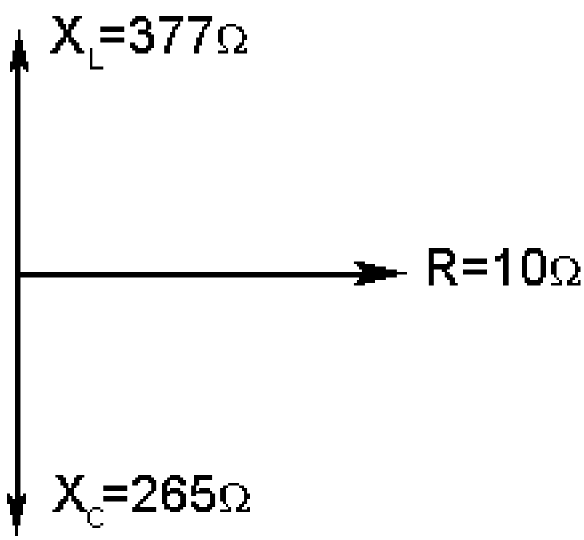
$Z = \sqrt{R^2 + X^2} = \sqrt{10^2 + (350.5)^2} = 350.6$

The angle formed by the phasor impedance with the reference (R) is calculated by means of

$\tan \phi = 350.5/10 = 35.05$

$\phi = \arctan 35.05 = 88.36^o$

a)



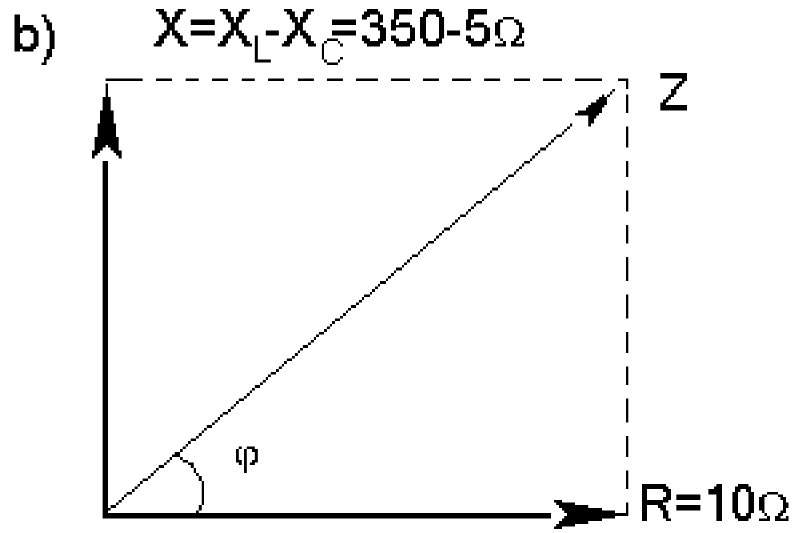


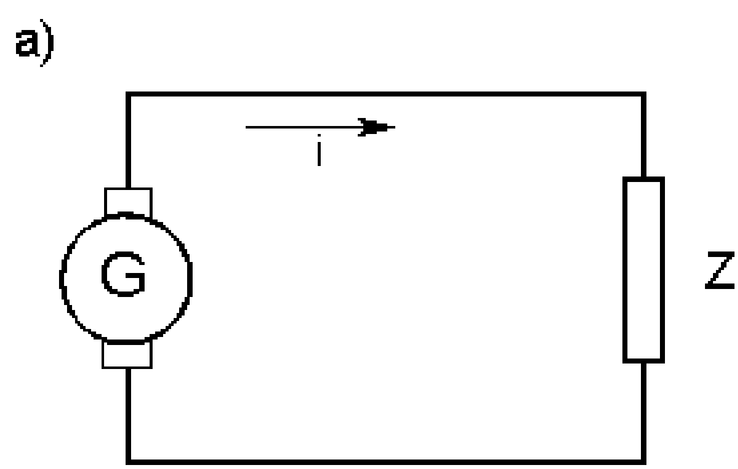
Figure 1.14. Phasor Representation.

a) Phasor diagram for each element

b) Impedance representation

Figure 1.15 a), presents the equivalent circuit substituting the RLC elements by it equivalent impedance and in b) the phasor representation for v and i is presented.

The Ohm' Law is valid for the AC system also. The basic difference is due to the mathematical complication because the calculations are not made with scalar quantities but with complex numbers.



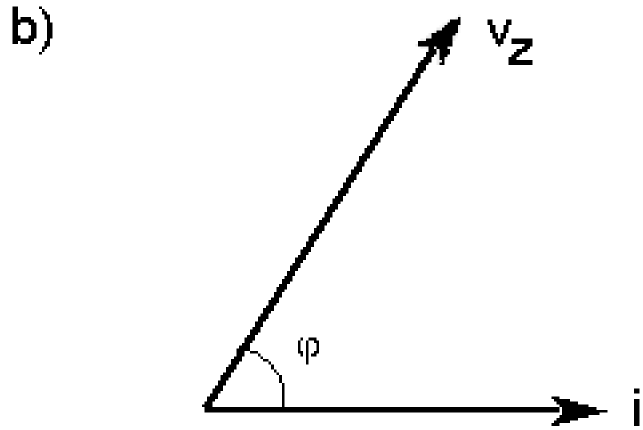


Figure 1.15. a) Equivalent Circuit; b) Phasor Representation

The voltage drop across the impedance is equal to the generated voltage, and given by

$$v_Z = i \; Z$$

The numerical value for I_{max} is

I_{max} = 141/350.6 = 0.4 Amp

As can be noted from Figure 1.16 b, the voltage leads the current in 88.36º . Is the same to say that the current lags the voltage in the same quantity. From all this, the equation for the current will be

$$i = 0.4 \sin (377t - 88.3^o) \text{ Amp}$$

As can be seen from Figure 1.15 b), The voltage drop across the impedance is composed of two voltage drops: one across R, expressed as v_R , in phase with the current i, and the other across the resultant reactance, expressed as v_X , 90º out of phase with v_R and i. These voltages and currents can be expressed by it RMS values, which are represented with the capital letters V, and I.

The power calculated using the impedance Z is known as apparent power (S), and its magnitude is given by

$$S = (V_z)(I)$$

Where V_z and I are the RMS values of the voltage and current respectively.

Applying the Ohm's Law: $I = V_z/Z$,

From which is obtained $S = (V_z)^2/Z$

The unit for measuring this power is volt-ampere (VA). The power obtained using the reactance X is numerically expressed as

$$Q = (V_x)(I) = (V_x)^2/X$$

Is the reactive power, and measured in volt-ampere reactive (VAR). At last, the power dissipated in the resistance is the average real power, measured in Watts with magnitude

$$P = V_R I = V^2/R$$

All This can be represented graphically in Figure 1.16.

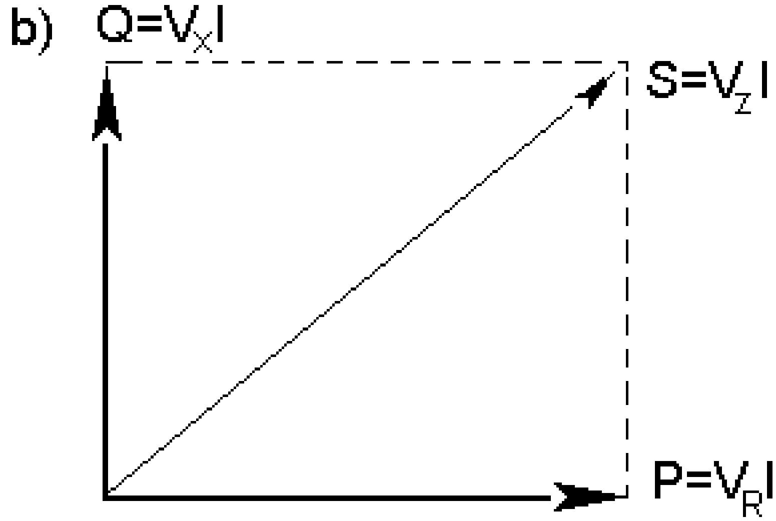


Figure 1.16. Power Representation

In AC electrical systems, the useful power is the average power. It is convenient to diminish the reactive power because this component has a negative effect in the process. The energy given to the system in one half of the cycle is extracted in the other half. This has the effect of a circulating current that is not given energy to the system. As can be seen from Figure 1.19, the smaller the reactive power Q, the smaller the angle j for the same apparent power S. The number given by cos j is known as the power factor. In industrial facilities where there are installed many motors (inductive loads) it is frequently necessary to place capacitors in parallel with the load in order to correct the power factor.

From Figure 1.16 can also be seen that when the reactive load does not exist, the apparent power S is equal to the average power P. Because of this, in pure resistive circuits it is possible to express the power dissipated by the load in volt-amperes (VA) or in Watts (W).

Example 1.7

Calculate the apparent, reactive, and average power for the problem in example 1.6.

The RMS value of the apparent power can be obtained from the voltage and current RMS values as follows

$V_{RMS} = V_{max}/\sqrt{2} = 141/1.41 = 100$ Volts

$I_{RMS} = I_{max}/\sqrt{2} = 0.4/1.41 = 0.28$ Amp

$S = (100)(0.28) = 28$ VA

For this particular case, the angle between the voltage across the impedance and the current is 88.3º so the power maybe represented by the Figure 1.17.

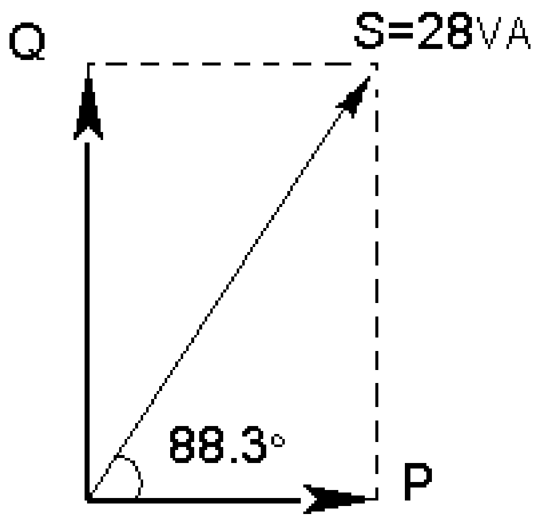


Figure 1.17. Power representation for S = 28 VA and $\phi = 88.3^o$

The reactive and average power are calculated using elementary trigonometric rules as:

$Q = S \sin \phi = 28 \sin 88.3^o = 28 \times 0.999 = 27.98$ VAR

$P = S \cos \phi = 28 \times 0.03 = 0.83$ W

1.8. RESIDENTIAL WIRING APPLICATION OF AC CIRCUITS

As an example let's study the way a common residence receives the electric power from the local power company. The electrical energy comes in through three wires originated from a transformer, and consists of a neutral wire, which is connected to earth ground, and two “hot” wires. Each of the hot lines supplies 120 V RMS to the dwelling, and are 180^o out of phase. According to the *NEC*, These two hot wires shall be one red, and one black. The neutral wire will be white. Figure 1.18 show the schematic representation of the voltage that enters the residence. The voltage difference between the two hot wires is given by:

$V_R – V_B$ = 120 – (-120) = 240 Volts.

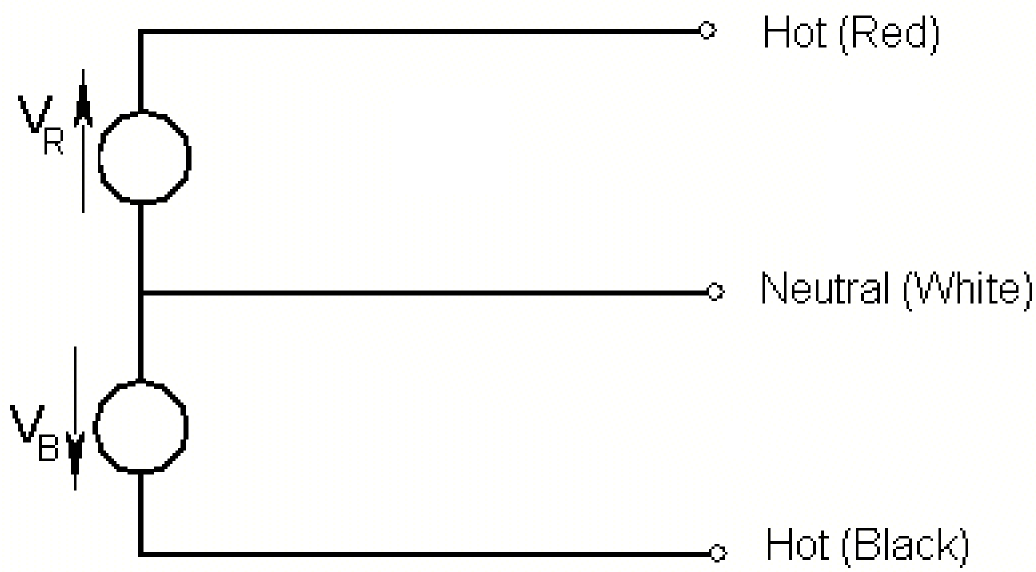


Figure 1.18. Voltage Entrance for Residential Circuits.

The lighting circuits, small appliances, and electronic devices are connected between one hot and the neutral wire. They receive 120 Volts. Appliances such as air conditioners, heaters, and ranges are connected between the two hot wires and receive 240 Volts. The reason for this is that appliances that require a considerable amount of power to operate will need also a relatively big current as can be understood from the basic power relation

$$P = VI \quad \text{or} \quad I = P/V$$

Increasing the voltage from 120 Volts to 240 Volts, the current is reduced to half its original value with the consequent reduction in the wire size for the same power. The reduction in wire size is convenient because it simplifies the house wiring, and smaller wires cost less. All this reduces the costs of the electrical wiring.

At this point could be raised a question. If higher voltage reduce the costs, why not to employ, for example 480 Volts in residential wiring? The reason for the use of a maximum of 240 Volts in residence circuits is safety. As higher the voltage, higher is the risk of being killed by an electric shock.

1.9. REVIEW QUESTIONS

1.1. - The unit of voltage is _____________________.

1.2. - The unit of current is _____________________.

1.3. - The unit of resistance is ___________________.

1.4. - The voltage drop in a resistance is 2.0 V and the current that circulates through it is 5 Amp. Calculate the resistance in ohms.

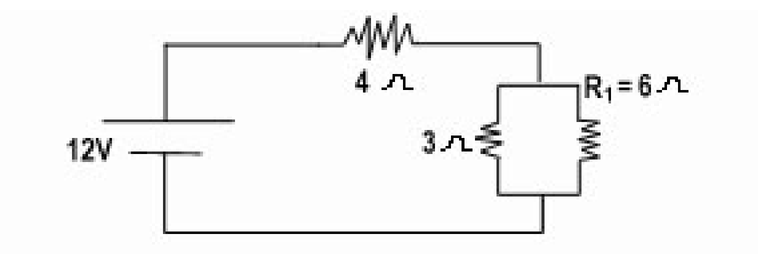
1.5. - Two resistances 3 Ω and 6 Ω are connected in parallel. Calculate the equivalent resistance.

1.6. - Three resistances 5 Ω, 7 Ω and 9 Ω are connected in series. Calculate the equivalent resistance.

1.7. - Calculate the resistance of 100 feet of copper wire (ρ = 10.5 cm/l/ft) and 0.01 inch in diameter.

1.8. - The resistance of a copper wire (α = 0.0023) at 32°F is 6.3 ohms ?. What will be its resistance at 90°F?

1.9. - Calculate the current through resistance R1 in the shown circuit.



1.10. - For a sinusoidal wave, the maximum voltage is 155.6 volts. Calculate the RMS voltage.

1.11. - The capacitance is measured in __________________.

1.12. - The inductance is measured in ___________________.

1.13. - The reactance is measured in ____________________.

1.14. - For $V = 300 \sin 628t$ and $R = 10\ \Omega$, find

a) V_{max}

b) V_{dc}

c) V_{RMS}

d) frequency

e) expression for i

f) I_{max}

g) I_{RMS}

1.15). -In a series RCL circuit, R = 100 Ohm, L = 0.1 Henry, and C = 10 microfarad. Calculate the apparent, reactive, and average power for a voltage drop across the three elements given by $v = 169 \sin 377t$.